which yields the constant C. When $\varkappa=-2$, the requirement represents the second condition of solvability. The solutions obtained when both conditions of solvability are satisfied, are isolated.

Since the condition $1-c^2>0$ corresponds to a weak, and $1-c^2<0$ to the strong shock wave /1/, it follows from the above analysis that when the wedge surface is subjected to an arbitrary conical perturbation, a solution exists and is unique if the unperturbed shock wave is weak, and there are no solutions if the shock is strong.

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TWO-VELOCITY MECHANICS OF GRANULAR POROUS MEDIA*

N.D. MUSAYEV

A two-phase mixture of a porous or granular solid phase and liquid or gas filling the pores or the intergranular spaces, is considered. Two limiting structures of the mixture are specified: 1) the solid phase represents dense packing of spherical particles (grains) in intergranular point contact; 2) the pores represent channels, almost cylindrical in form. Expressions for the interphase forces and equations of the two-velocity motion of the phases are studied within these two structures. Different development of the interphase forces depending on the structure of the mixture is noted, the forces arising from the forces of inertia and in particular from the Archimedes and the attached-mass forces.

Using the representations of the multivelocity continuum, we shall write the equation of conservation of phase masses in the form /1/

$$\partial\rho_1/\partial t+\nabla^k\rho_1v_1^k=J_{21},\quad \partial\rho_2/\partial t+\nabla^k\rho_2v_2^k=J_{12} \tag{1}$$
$$(\rho_i=\rho_i^\circ\alpha_i,\ i=1,\ 2;\ \alpha_1+\alpha_2=1)$$

The lower indices $i=1,2$ refer, respectively, to the parameters of the liquid (gaseous) and solid phases, ρ_i^0 and ρ_i are the real and apparent density connected with each other through the volume concentration, α_i, v_i is the velocity of the i-th phases, and J_{ji} is the phase transition intensity characterizing the amount of mass of the j-th phase transported to the i-th phase per unit volume of the mixture, in unit time $(i,j=1,2;\ i\neq j)$.

The equations of the phase moments can be written in the form /1/

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$$\rho_1 \frac{d_1 v_1{}^k}{dt} = \nabla^l \sigma_1{}^{lk} - R_{12}^k - J_{12}(v_2{}^k - v_1{}^k) + \rho_1 g_1{}^k \tag{2}$$
$$\rho_2 \frac{d_2 v_2{}^k}{dt} = \nabla^l \sigma_2{}^{lk} + R_{12}^k + \rho_2 g_2{}^k$$

Here $\sigma_i{}^{lk}$ is the stress tensor in the i-th phase, defining the transfer of momentum in the i-th phase across the unit plane cross section in the mixture, $R_{12}{}^k$ is the interphase force determining the transfer of momentum from the first to the second phase across the interphase surface per unit volume of the mixture, and $g_i{}^k$ are the external mass force vectors. When writing the term connected with the transfer of momentum caused by phase transitions, we assumed that the mass undergoing a phase transition has a velocity identical with the velocity of the solid (second) phase. Moreover, when $\sigma_1{}^{lk}$ and $\sigma_2{}^{lk}$ are given, we shall neglect the transfer of momentum caused by the pulsatory (small scale) motion of the phases.

If we regard the porous phase as a dense packing of spherical dispersed particles (grains) in point contact, we can specify $\sigma_1{}^{lk}$, $\sigma_2{}^{lk}$ and $R_{12}{}^{lk}$ using the representations which were formulated for the disperse mixtures. To do this we can introduce the reduced stress tensors in the phases $\sigma_{1*}{}^{lk}$ and $\sigma_{2*}{}^{lk}$ which include, apart from the stresses $\sigma_1{}^{lk}$ and $\sigma_2{}^{lk}$, the interphase interaction along the interphase surfaces of the particles dS_{21s} cut off by the plane cross-section dS^l and adjacent to it /1/

$$\sigma_{1*}^{lk} = \sigma_1^{lk} + \sigma_{21s}^{lk}, \quad \sigma_{2*}^{lk} = \sigma_2^{lk} + \sigma_{12s}^{lk}, \quad \sigma_{21s}^{lk} = -\sigma_{12s}^{lk} \tag{3}$$

where $\sigma_{21s}{}^{lk}$ and $\sigma_{12s}{}^{lk}$ are the interphase stress tensors. If we neglect the effect of viscosity in the stress tensors, assuming that it can manifest itself only through the interphase force, then we can adopt the following relations for the mixture with particles in point contact:

$$\sigma_{1*}^{lk} = -p_1\delta^{lk}, \quad \sigma_1^{lk} = -\alpha_1 p_1 \delta^{lk}, \quad \sigma_{21s}^{lk} = -\sigma_{21s}^{lk} = -\alpha_2 p_1 \delta^{lk} \tag{4}$$
$$\sigma_2^{lk} = -\alpha_2 p_1 \delta^{lk} - \sigma_{2*}^{lk}$$

and write the principal vector of the surface forces in the phases in the form

$$\nabla^l \sigma_1^{lk} - R_{12}^k = \nabla^l \sigma_{1*}^{lk} - nf = -\nabla^k p_1 - nf \tag{5}$$
$$\nabla^l \sigma_2^{lk} + R_{12}^k = \nabla^l \sigma_{2*}^{lk} + nf = \nabla^l \sigma_{2*}^{lk} + nf, \quad n = \frac{3\alpha_2}{4\pi a_2{}^3}$$

Here f is the interphase force acting from the direction of the liquid or gas, arriving at a particle or a granule whose whole surface, excpet for several points of contact, is washed by the liquid or gas, n is the numerical concentration of the particles, and a_2 is their radius. The stress tensor in the mixture is equal to

$$\sigma^{lk} \equiv \sigma_1{}^{lk} + \sigma_2{}^{lk} \equiv \sigma_{1*}{}^{lk} + \sigma_{2*}{}^{lk} = -p_1\delta^{lk} + \sigma_{2*} \tag{6}$$

The above expressions imply that the reduced stress $\sigma_{2*}{}^{lk}$ is defined in terms of directly measurable quantities, namely the total stress in the mixture σ^{lk} and the pressure within the pores p_1, and can be interpreted as a part of the stress tensor $\sigma_2{}^{lk}$ in the solid phase governed by the transfer of momentum through the particle contacts, independent of the liquid or gas.

In the disperse mixtures of low concentration, where there are no such contacts, we have $\sigma_{2*}{}^{lk} = 0$.

Remembering that the whole surface of the dispersed particles (except for the finite number of the interparticle point contacts) is surrounded by the liquid (gas), we can represent the interphase force per single particle /1/ in the form of the sum of the Archimedes force f_{A1}^k, attached masses $f_m{}^k$ and the Stokes-type viscous friction force $f_\mu{}^k$, while the neglecting the Basset-type force), since the "viscous boundary layer" surrounding the particle is not stationary). Then the equations of phase moments and the total force of interphase interaction $R_{12}{}^k$ can be rewritten in the form

$$\rho_1 \frac{d_1 v_1{}^k}{dt} = \alpha_1 \nabla^k p_1 - F_m^k - F_\mu{}^k - \alpha_1 J_{12}(v_2{}^k - v_1{}^k) + \rho_1 g^k \tag{7}$$
$$\rho_2 \frac{d_2 v_2{}^k}{dt} = \alpha_2 \nabla^k p_1 - \nabla^k \sigma_{2*}^k - F_\mu{}^k + \alpha_2 J_{12}(v_2{}^k - v_1{}^k) - \rho_2 g^k$$
$$F_m{}^k = \alpha_2 n f_m{}^k = \frac{1}{2}\eta_m \alpha_1 \alpha_2 \left(\frac{d_1 v_1{}^k}{dt} - \frac{d_2 v_2{}^k}{dt}\right)$$
$$F_\mu{}^k = \alpha_1 n f_\mu{}^k = \eta_\mu \alpha_1 \alpha_2 \mu_1 a_2{}^{-2} (v_1{}^k - v_2{}^k)$$
$$R_{12}{}^k = \nabla^k \alpha_2 p_1 + n f_{A1}^k - n f_m{}^k + n f_\mu{}^k = p_1 \nabla^k \alpha_2 - F_m{}^k - F_\mu{}^k +$$
$$\alpha_2 J_{12}(v_2{}^k - v_1{}^k)$$

Here μ_1 is the viscosity of the first phase, η_m and η_μ are coefficients expressing the effect of the particle interaction.

Let us now consider another structure of a porous medium saturated with a liquid which fills the pores of almost cylindrical, channel-like shape. In the case of such a structure the tensors $\sigma_{12s}^{lk}, \sigma_{21s}^{k}$, the force f^k and the numerical concentration of the particles lose their sense and the stresses (3)-(7) cannot be utilized. In this case we define the stress in the liquid phase in terms of the pressure, following the reasoning used in (4). As in (6), we introduce the reduced stress tensor in the solid phase $\sigma_{2*}{}^{lk}$

$$\begin{gathered}\sigma_1{}^{lk} = -\alpha_1 p_1 \delta^{lk}, \ \sigma_2{}^{lk} = -\alpha_2 p_1 \delta^{lk} + \sigma_{2*}{}^{lk} \\ \sigma = \sigma_1{}^{lk} + \sigma_2{}^{lk} = p_1 \delta^{lk} + \sigma_{2*}{}^{lk}\end{gathered} \tag{8}$$

We see that the stresses σ_{2*}/α_2 characterize the difference between the mean stresses $\sigma_2{}^{lk}/\alpha_2$ in the solid phase and the pressure within the pores. As earlier in (6), the stress $\sigma_{2*}{}^{lk}$ can be found from the measurements of σ^{lk} and p_1 /2/.

If the channels in the porous medium are smooth, rectilinear and oriented along the direction of relative acceleration of the phases, then R_{12} does not contain the term $\Delta R_{12}^{(r)}$ since the pressure oscillations are small scale. A term $\Delta R_{12}^{(\tau)}$, however, appears due to the viscous friction between the liquid and the channel walls

$$\begin{gathered}R_{12}{}^k = p\nabla^k \alpha_2 + \Delta_{12}^{(\tau)k}, \quad \Delta R_{12}^{(\tau)k} = F_\mu{}^k \\ F_\mu{}^k = \eta_\mu a_*^{-2} \mu_1 \alpha_1 \alpha_2 (v_1{}^k + v_2{}^k)\end{gathered} \tag{9}$$

where a_* is the pore radius. In the general case, when the channels are curved, a component $\Delta R_{12}^{(p)k}$ appears, caused by the inertial interaction between the phases

$$\begin{gathered}\Delta R_{12}^{p)} = F_m{}^k = \eta_m \left[\frac{1}{2}\alpha_1\alpha_2\rho_1{}^0\left(\frac{d_1 v_1{}^k}{dt} - \frac{d_2 v_2{}^k}{dt}\right)\right] + \\ \alpha_2 J(v_2 - v_1); \quad 0 \leqslant \eta^m \lesssim 1\end{gathered} \tag{10}$$

As a result, the equations of phase moments (7) generalized to the porous and granular media, have the form

$$\begin{gathered}\rho_1 \frac{d_1 v_1}{dt} = -\alpha_1 \nabla p_1 - F_m - F_\mu - J_{12}(v_2 - v_1) + \rho_1 g_1, \\ \rho_2 \frac{d_2 v_2}{dt} = -\alpha_2 \nabla p_1 + \nabla^k \sigma_{2*}^{k} + F_m + F_\mu - \rho_2 g_2\end{gathered} \tag{11}$$

The conservation equations (1) and (11) are closed by the equations of state for the liquid phase and the equations of state for the porous phase, namely by the equations for $\sigma_{2*}{}^{lk}$. A variant of the thoery describing the elastic behaviour of the porous phase is given in /1, 2/.

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